HIGH ORDER FLUID MODEL FOR STREAMER DISCHARGES

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Abstract. A high order fluid model for streamer discharges is developed and used to investigate propagation of negative streamer fronts in N_2. Momentum transfer theory is employed to evaluate the collision terms and close the system of moment/balance equations. The results of simulations are compared with those obtained by a PIC/MC method and by the classical first order fluid model based on the drift-diffusion and local field approximations. The comparison clearly validates the theoretical basis of the high order fluid model, while the first order fluid model underestimates many aspects of streamer dynamics.

1. INTRODUCTION

When a strong electric field is applied to non-ionized or lowly ionized matter, filaments of weakly ionized non-equilibrium plasma, called streamers can grow. They can be observed in different kinds of high-pressure discharges, for example in corona discharges, but they also appear in the form of plasma bullets observed in plasma jets [1]. Streamers have applications in diverse areas of science and technology ranging from their role in creating lightning and transient luminous events in the upper atmosphere [2] to industrial applications such as the treatment of polluted gases and water [3].

During the last three decades, the prevalent opinion has been that streamer dynamics could be described adequately by the so-called first order fluid model based on the drift-diffusion and local field approximations for all species in the plasma-including the electrons. However, recent modeling [5] suggests that we should revise this opinion. In typical situations (e.g., in the pure gases and little explicit effects of photo-ionization) the local field approximation is generally insufficient to represent the electron dynamics as the electron energy depends upon the electric field in a wider spatial range. One way to deal with this issue has been recently demonstrated by Li *et al.* [6] through the development of the so-called extended fluid

models. These models involve a density gradient expansion of the source term to approximate the spatial non-locality of the ionization processes at the streamer front. The alternative way to incorporate the complex electron dynamics in the streamer front is to consider the energy balance equation. In this work we illustrate that this is not a straightforward process and we show how to derive a consistent set of fluid equations beyond the equation of continuity and the momentum balance equation.

2. THEORY

The starting point of our formalism is a set of moment/balance equations which is found by multiplying the Boltzmann equation by an arbitrary function $\Phi(\boldsymbol{c})$ of the charged particle velocity, and integrating over all velocities

$$\frac{\partial}{\partial t}\Big[n\Phi(\boldsymbol{c})\Big] + \nabla\cdot\Big[n\langle\boldsymbol{c}\Phi(\boldsymbol{c})\rangle\Big] - n\frac{q}{m}\langle\boldsymbol{E}\cdot\frac{\partial}{\partial c}\Phi(\boldsymbol{c})\rangle = -\int\Phi(\boldsymbol{c}J(f))d\boldsymbol{c}\,, \quad (1)$$

where $\langle\,\rangle$ represents the average over particle velocity $\boldsymbol{c}$, q and m are the charge and mass of the charge particle, $\boldsymbol{E}$ is the electric field vector and J is the collision operator accounting for elastic and inelastic collisions. If one takes $\Phi(\boldsymbol{c})$ equal to 1, $m\boldsymbol{c}$, $\frac{1}{2}mc^2$ and $\frac{1}{2}mc^2\boldsymbol{c}$, etc., one generates an infinite series of equation, a full solution of which would be equivalent to calculating the phase space distribution function $f(\boldsymbol{r},\boldsymbol{c},t)$. In practice, however, one must truncate the chain, and in this work we propose that the energy flux equation is crucial for the success of the fluid model of streamer discharge:

$$\frac{\partial n}{\partial t} + \nabla\cdot n\boldsymbol{v} = \quad C_0\,, \quad (2)$$

$$\frac{\partial}{\partial t}\Big[nm\boldsymbol{v}\Big] + \nabla\cdot\Big[nm\langle\boldsymbol{cc}\rangle\Big] = \quad C_{m\boldsymbol{c}}\,, \quad (3)$$

$$\frac{\partial}{\partial t}\left(n\varepsilon\right) + \nabla\cdot\left(n\boldsymbol{\xi}\right) - nm\boldsymbol{a}\cdot\boldsymbol{v} = \quad C_{\frac{1}{2}mc^2}\,, \quad (4)$$

$$\frac{\partial}{\partial t}\left(n\boldsymbol{\xi}\right) + \nabla\cdot\Big[n\Big\langle\frac{1}{2}nmc^2\boldsymbol{cc}\Big\rangle\Big] - nm\boldsymbol{a}\Big\langle\frac{\partial}{\partial\boldsymbol{c}}\left(\frac{1}{2}mc^2\boldsymbol{c}\right)\Big\rangle = \quad C_{\frac{1}{2}mc^2\boldsymbol{c}}\,, \quad (5)$$

where $\boldsymbol{v} = \langle\boldsymbol{c}\rangle$, $\boldsymbol{a}$ is the acceleration due to the electric field force, ε is the mean energy and $\boldsymbol{\xi}$ is the energy flux.

The fluid equations (2)-(5), obtained as velocity moments of the Boltzmann equation are closed in the local mean energy approximation and coupled to the Poisson equation for the space charge electric field. The high order tensors appearing in the energy flux equation are specified in terms of lower order moments. The collision terms are evaluated using the momentum transfer theory, and are given by:

$$C_0 = -n\left(\nu_A - \nu_I\right)\,, \quad (6)$$

$$C_{m\boldsymbol{c}} = \frac{mm_0}{m+m_0} n\nu_m \boldsymbol{v} - nm\boldsymbol{v}\left[\nu_A - \zeta\nu_A'\right], \tag{7}$$

$$C_{\frac{1}{2}mc^2} = -n\nu_e\left(\varepsilon - \frac{3}{2}kT\right) - \frac{m_0}{m+m_0}\sum_i \left(\nu_i - {\nu_i}^s\right)\epsilon_i - n\varepsilon\nu_A - n\nu_i^{(i)}\Delta\varepsilon_i\,, \tag{8}$$

$$C_{\frac{1}{2}mc^2\boldsymbol{c}} = -n\nu_m\boldsymbol{\xi}\,, \tag{9}$$

where ν_m and ν_e are the average collision frequencies for momentum and energy transfer, ν_A and ν_I are the attachment and ionization rates, while ζ is given by

$$\zeta = \frac{2}{3}\frac{m_0}{m_0+m}\left[\frac{1}{2}m\langle c^2\rangle - \frac{1}{2}m\boldsymbol{v}^2\right]. \tag{10}$$

Likewise, ν_i and ${\nu_i}^s$ are inelastic and superelastic collision frequencies for inelastic channel i while ϵ_i and $\Delta\epsilon_i$ are the thresholds for inelastic and ionization processes, respectively.

3. RESULTS AND DISCUSSION

The finite volume method is used to spatially discretize the system (2)-(5) on a uniform grid with 1000 points. To approximate the spatial derivative we use the second-order central difference discretization while the time derivatives are approximated with the Runge-Kutta 4 method. The continuity equation for the electron and ion densities has a second order spatial derivative, and therefore requires two boundary conditions for each direction in space. For $x = 0$ we use Neumann boundary condition, so that electrons that arrive at those boundaries may flow out of the system. For $x = L$ we employ Dirichlet boundary condition to ensure that there is no outflow of electrons from the system. In all calculations we set $L = 1.2$ mm.

The average collision frequencies for momentum and energy transfer in elastic and inelastic collisions required as an input in the fluid equations are calculated using a multi term Boltzmann equation solution [7]. The cross sections for the electron scattering in N_2 detailed by Stojanović and Petrović [8] are used in this work. The results of simulations are compared with those obtained by a PIC/MC method [5, 6] and by the classical first order fluid model based on the drift-diffusion and local field approximations.

In figure 1 we show the electron density and mean energy for electrons after 1 ns. Our results and those obtained by a PIC/MC method agree very well. High-order fluid profiles are slightly faster than those obtained by the PIC/MC method but the agreement is much better than between the PIC/MC results and those obtained by the first order fluid model based on the drift-diffusion approximation, particularly for the ionization level behind the front.

Perhaps one of the most striking properties is the behavior of the mean energy in the streamer channel. Although the electric field is entirely screened inside of the channel, the mean energy significantly exceeds thermal energy. This is a typical non-local effect as the mean energy does not have enough time to be fully thermalized in the streamer interior on the time

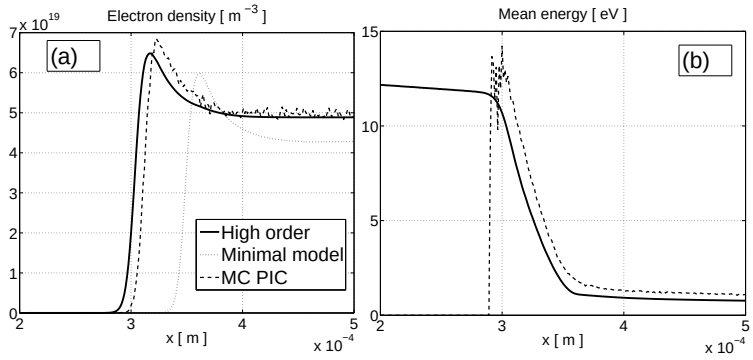

Figure 1. Electron density (a) and mean energy (b) after 1 ns. The simulation is started with the same initial Gaussian distribution for electrons and ions with a maximum density of 2×10^{18} m^{-3} at the position $x = 8 \times 10^{-4}$ m from the left boundary.

scale relevant for streamer formation under conditions considered in this work. The lower ionization density behind the front in the classical fluid model is also the effect of too low electron energies, this time in the streamer head.

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